CALCULATION OF FLOW NEAR THE CRITICAL POINT WITH LIQUID COOLING

I. G. Druker and L. Ya. Treer

UDC 533.601.155;536.246:532.526

The problem of thermal protection is formulated exactly and solved for objects covered with a film of liquid coolant. The results of calculations are shown for an axially symmetrical object in a stream of oxygen.

Formulation of the Problem. During the aerodynamic heatup of an object flying at supersonic speed, a liquid film may be formed on its surface either naturally (surface melting) or by artificial means (if a liquid coolant is supplied). The result is a two-layer flow: the outer layer consists of the oncoming gas and liquid vapors; the inner layer is a thin liquid film.

Such phenomena have been studied in [1-3]. In these references, however, the film flow was treated separately from the gas flow, the friction stress at the gas–liquid boundary was determined from the stream pattern around the solid object, the effect of evaporation on the heat dissipation was considered approximately only, the inertia terms in the equation of film flow were disregarded, etc.

Although simplifications are entirely justified in several cases, it is worthwhile to calculate the liquid layer and the gas layer exactly, taking into account the interaction between them.

With these considerations in mind the authors have in [4] analyzed the problem of surface melting for the case of an axially symmetrical vitreous object flying at a high speed. Only one assumption was made there, namely, that surface melting and wear did not change the basic shape of the object. This holds true when the supply of liquid coolant is distributed over the surface, and this case will be studied here. The flow rate of the coolant will here be an independent parameter, unlike in [4], where it was determined uniquely by the stream conditions. Due to peculiar thermophysical properties of the vitreous material, furthermore, the thickness of the liquid layer is formally infinite but in this case precisely defined.

The equations given here subsequently describe the flow both in the gas and in the liquid layer around an immersed object near the critical point. The origin of the coordinate normal to the object surface lies on the gas–liquid boundary and the positive direction is into the gas.

The Equations. The flow of a gas near the critical point is described by the equations of a supersonic boundary layer in the Liz-Dorodnitsyn variables [5-7]:

the flow equation

$$(lf_{\lambda\lambda})_\lambda + ff_{\lambda\lambda} + \frac{1}{2}\left(\frac{\rho_l}{\rho} - f_\lambda^2\right) = 0 ;$$

the energy equation

$$\left(\frac{\gamma l}{\mathrm{Pr}} t_\lambda\right)_\lambda + t_\lambda\left(f\gamma - \frac{m}{R}\sum_{\alpha=1}^{\mu} C_{p\alpha}\Phi_\alpha m_\alpha\right) = 0 ;$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 44-48, March-April, 1972. Original article submitted July 12, 1971.

© 1974 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $15.00.

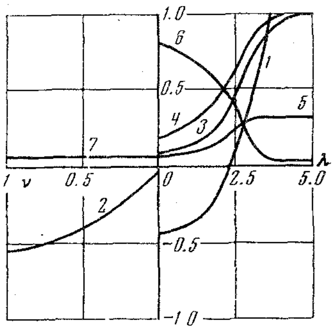

Fig. 1

the equations of diffusion referred to the components

$$(\Phi_\alpha)_\lambda - fc^*_{\alpha\lambda} = 0 \qquad (\alpha = 1, 2 \ldots \mu);$$

the equation of state

$$P = \rho \frac{R}{m} T .$$

The equations of diffusion and the energy equation are written here for a so-called frozen flow in the boundary layer.

The diffusion currents of all components are related to their concentrations as follows [8]:

$$\Phi_\alpha = \frac{l}{\mathrm{Pr}} \sum_{\beta=1}^{\mu} m_\beta L_{\alpha\beta} \left(c^*_{\beta\lambda} \sum_{\alpha=1}^{\mu} c_\alpha{}^* - c_\beta{}^* \sum_{\alpha=1}^{\mu} c^*_{\alpha\lambda} \right)$$

The following dimensionless functions have been introduced here:

$$\Phi_\alpha = \frac{I_\alpha \rho}{\lambda_n \rho_0 \eta_0}, \qquad \Upsilon = \frac{c_p m}{R}$$

$$\lambda_n = \frac{\partial \lambda}{\partial n} = \frac{r \rho u_e}{\sqrt{2\xi}}, \qquad l = \frac{\eta \rho}{\eta_0 \rho_0}, \qquad t = \frac{T}{T_e}$$

with m denoting the molecular weight of any one arbitrarily chosen component, I_α the diffusion current of component α in the gas mixture, C^*_α the mass concentration of component α referred to its molecular weight, and $L_{\alpha\beta}$ the generalized Lewis coefficients; all other designations are the conventional ones.

The flow of liquid in the film is described by the equations of an incompressible boundary layer, where the viscosity can be highly temperature dependent. For this reason, transformations analogous to the Liz-Dorodnitsyn transformations were applied to the liquid layer as well. Unlike in the gas layer, where the parameters are referred to values at its outer boundary, the new variables here are referred to values at the gas–liquid boundary:

$$\nu = \frac{r u_0}{\sqrt{2\zeta}} \int_n^0 \rho_\varkappa dn, \qquad \zeta = \int_0^s \rho_\varkappa \eta_\varkappa u_0 r^2 ds$$

where subscript $\varkappa$ refers to properties of the liquid layer.

The equations for the liquid layer become now:

the flow equation

$$(l_\varkappa \psi_{\nu\nu})_\nu + \psi\psi_{\nu\nu} + \frac{1}{2}\left(\frac{\rho_l}{\rho_\varkappa}\frac{1}{f_{\lambda_0}{}^2} - \psi_\nu{}^2\right) = 0 ; \qquad (1)$$

the energy equation

$$\left(\frac{k_\varkappa}{c_{p\varkappa}\eta_\varkappa} t_\nu\right)_\nu + \psi t_\nu = 0 .$$

The Boundary Conditions. The usual conditions apply to the outer boundary of the gas layer ($\lambda \to \infty$) determined by the outer stream around the object. At the liquid–solid boundary one may equate to zero the longitudinal component of velocity and the temperature.

The conditions at the gas–liquid boundary require a more careful consideration.

Some of the conditions here follow from the continuity of flow variables: temperature, tangential velocity, frictional stress, and transverse flow of mass and energy. The conditions for the concentrations

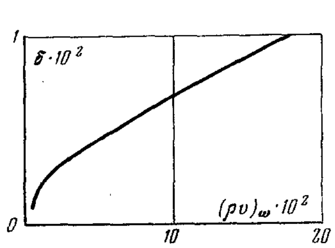

Fig. 2

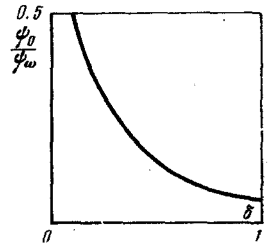

Fig. 3

of the individual components follow from the law of conservation, with no chemical reactions assumed to be occurring at the boundary.

When the liquid evaporates, furthermore, the equation of evaporation kinetics must be given. Thus, when the boiling point is reached (at a given pressure), evaporation will be determined by the thermal flux from the gas layer.

When supplying a liquid coolant, one usually specifies the flow rate and determines the corresponding film thickness, for which an additional condition is required. In order to manage without this condition in our analysis, both the thickness of the liquid layer and the flow rate will be given here.

With all these considerations the boundary conditions in Liz-Dorodnitsyn variables become:

at the outer boundary of the gas layer ($\lambda \to \infty$)

$$t = 1, \quad f_\lambda = 1, \quad c_\alpha^* = c_{\alpha e}^* \qquad (\alpha = 1, 2 \ldots, \mu) ;$$

at the inner boundary of the film ($\nu = \delta$)

$$\psi_\nu = 0, \quad t = T_v / T_e .$$

Here T_V is the temperature at the solid wall and δ is the thickness of the liquid film.

At the gas–film boundary (continuity of tangential velocities, of shear stresses, of temperatures, and of transverse mass flow)

$$u = u_\varkappa, \quad t = t_\varkappa$$

$$f_{\lambda\lambda} = \sqrt{\frac{\rho_\varkappa \eta_\varkappa}{\rho\eta} \frac{1}{f}} f_\lambda{}^2 \psi_{\nu\nu}, \qquad \frac{f}{\psi} = \sqrt{\frac{\rho_\varkappa \eta_\varkappa}{\rho\eta}} f_\lambda$$

$$\frac{\gamma}{\Pr} t_\lambda + f \frac{L}{T_e R} = 0$$

where L is the heat of evaporation of the liquid coolant.

The condition which follows from the definition of $\psi_\nu = u/u_0$ is

$$\psi_\nu = 1$$

and the conditions for the concentrations are

$$c_\alpha^* f = \Phi_\alpha \qquad (\alpha = 1, 2 \ldots, \mu - \varepsilon)$$

$$c_\beta^* = \frac{\Phi_\beta}{f} + \frac{1}{m_\beta} \qquad (\beta = \mu - \varepsilon + 1 \ldots \mu)$$

where ε is the number of components entering the gaseous phase during evaporation.

The condition $u = u_\varkappa$ has been used in the derivation of Eq. (1) for eliminating the unknown velocity u_0.

Method of Calculation. The system of equations obtained here has been solved by the method of finite differences. The equations were written down in the difference form and then solved by the successive approximations.

A linear concentration and temperature profile with a quadratic flow function was taken as the zeroth approximation.

The system of difference equations was solved by the method of iterations, and then with the aid of the boundary conditions the next approximation was found for the boundary value.

Oscillations of the solution were avoided by means of the damping technique proposed in [6].

Results of Calculations and Discussion. For illustration we consider a solid object immersed in a stream of oxygen with water as the coolant.

Under the assumption that freezing will occur but no reaction between the coolant vapor and the oxygen, the gas layer consists of three components: O, H_2O, and O_2 (i.e., $\mu = 3$ and $\varepsilon = 1$).

The coolant is supplied in such a way that the liquid appearing on the surface of the solid object is at its boiling temperature. Consequently, the temperature gradient across the film may be assumed equal to zero.

Underheated water is not adequate for thermal protection because, as calculations have shown, the film flows off before evaporation becomes effective.

All calculations were performed for a sphere with a 1 m radius and with $u_{es} = 2200$ sec^{-1}, at a pressure $P_e = 1$ atm abs., $T_V = 373$°K, and $T_e = 6500$°K. The thermodynamic characteristics were taken from [9, 10].

The film thickness δ, varied from 0.01 to 2.0 (in Liz-Dorodnitsyn variables), was the controlling parameter.

The relation between the dimensionless variable ν and the normal in coordinates n is given as

$$n = \sqrt{\frac{\eta_x}{2u_{es} f_{\lambda 0}}}\, \nu .$$

The method could be used in a series of calculations for several thicknesses. The results have shown that varying the film thickness has little effect on the gas-layer characteristics but largely determines the relative rate of evaporation.

In terms of effectiveness, such a variant of thermal protection is preferable, since it allows one to utilize the effective heat capacity of the coolant to the fullest extent.

The typical distributions shown in Fig. 1 represent the dimensionless flow function for the gas (curve 1) and for the liquid film (curve 2), the referred temperature of the gas (curve 3), the concentrations of components in the gaseous boundary layer (c_0^* curve 4, $c_{O_2}^* \cdot 10^{-2}$ curve 5, and $c_{H_2O}^*$ curve 6), and the referred temperature of the liquid film (curve 7). The discontinuity between the flow functions for the gas and the liquid is due to the fact that each is referred to a different characteristic quantity.

In Fig. 2 we show how the thickness of the liquid film depends on the flow rate of the coolant. The subscript ω refers to the liquid–solid boundary. It is evident here that, when $\delta \to 0$, the flow rate tends toward a finite value. In this extreme case the entire supply of coolant evaporates, and no liquid film exists. This is clearly indicated in Fig. 3, where the relative fraction of evaporating coolant is plotted on a curve. As $\delta \to 0$, this fraction reaches 100%.

LITERATURE CITED

1. Bethe and Adams, "Theory of ablation applied to vitreous materials," Vopr. Raketn. Tekhn., No. 2 (1960).
2. G. Sutton, "The hydrodynamics and heat conduction in a melting surface," J. Aeronaut. Sci., 25, No. 1 (1958).
3. V. Ya. Borovoi, "Studies concerning the heating of a permeable surface in a supersonic gas stream with a supply of a variable-viscosity liquid," Trudy TsAGI, No. 1108 (1969).
4. I. G. Druker and L. Ya. Treer, "Thermal protection of apparatus by means of a flowing liquid surface film," in: 1st Siberian Conference on Aerogasdynamics (annotated reports), Sibirsk. Otdel. Akad. Nauk SSSR, Novosibirsk (1969).

5. V. V. Shchennikov, "Calculation of à laminar boundary layer at a sublimating surface," Zh. Vychislit. Matem. i Matem. Fiz., 1, No. 5 (1961).
6. B. V. Alekseev, "Laminar boundary layer at a sublimating surface," Zh. Vyshislit. Matem. i Matem. Fiz., 4, No. 3 (1964).
7. B. V. Alekseev, A Boundary Layer with Chemical Reactions [in Russian], Vychislit. Tsentr. Akad. Nauk SSSR, Moscow (1967).
8. J. Hirschfelder, C. Curtiss, and R. Bird, Molecular Theory of Gases and Liquids, Wiley (1964).
9. Thermodynamic Properties of Various Materials [in Russian], Izd. Akad. Nauk SSSR, Moscow (1962).
10. N. B. Vargaftik, Handbook on the Thermophysical Properties of Gases and Liquids [in Russian], Fizmatgiz, Moscow (1963).